MATH-5C TEST 1 v1 (Chapter 12, 13.1, 13.2, 13.4i)
Fall 24

100 points **NAME:**________________________

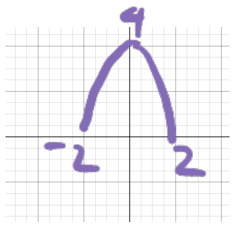
- **Full instructions on Canvas**
- **I expect clear and legible presentations with words of explanation. No credit given if work is not shown.**
- **Exact answers required unless otherwise stated.**
- **You are allowed one page of notes**
- **You may use graphing software to check your answers, but not to obtain the graphs for you.**

7 each

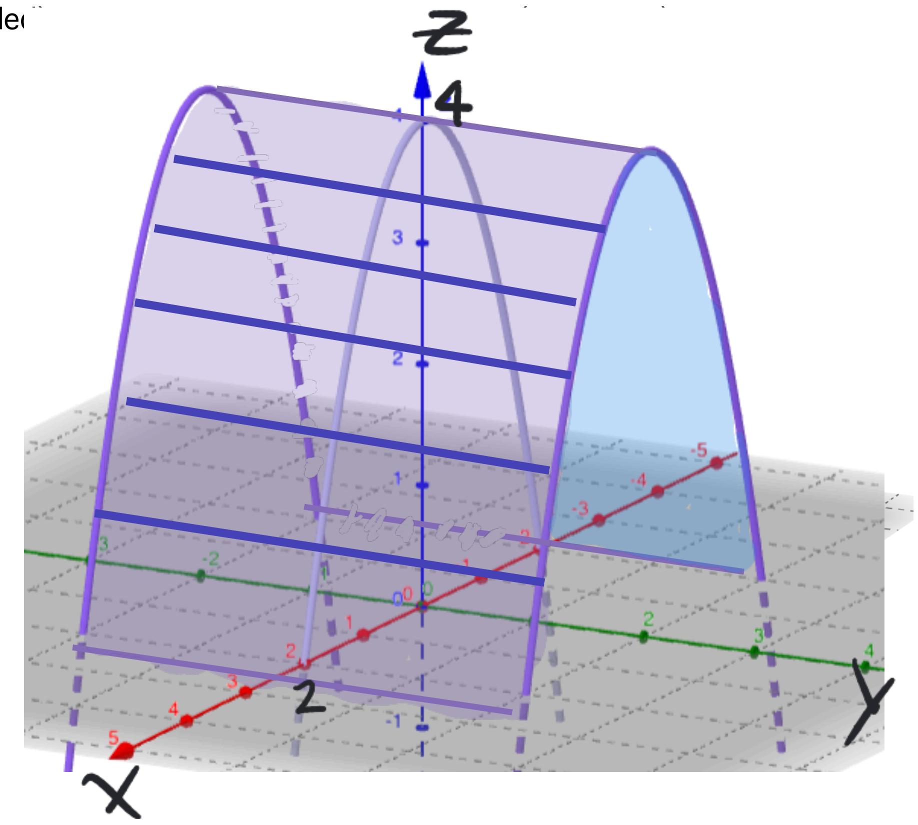
(1) On separate axes, sketch a graph of the following surfaces. Name the surface and give pertinent information such as traces. Show scale clearly on axes and one important cross section.(Use small grids for traces if neede

(a) $z = 4 - x^2$

Cylinder

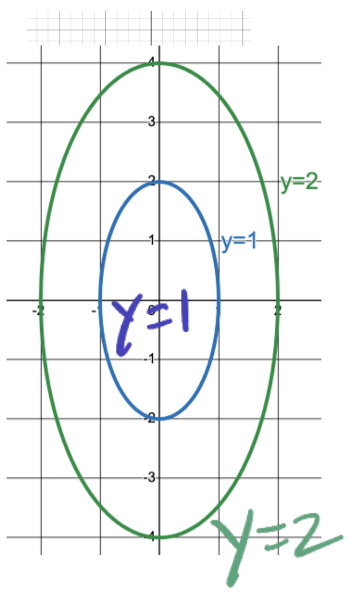


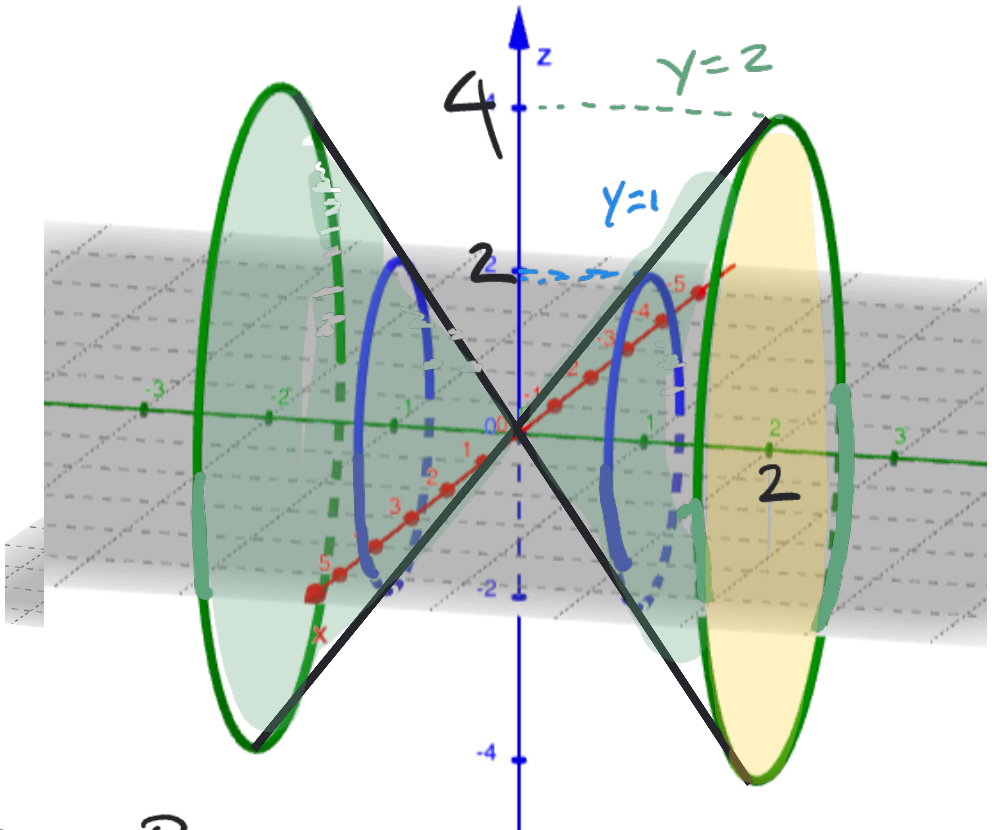
Rulings should be of same length and perallel to y axis.



(b) $y^2 = x^2 + \frac{z^2}{4}$

Cone





Note: Axes should be a Right Hand System (ask if not clear)

$r'(t) = \langle -2\pi \sin(2\pi t), 3t^2, \frac{t}{\sqrt{t^2+3}} \rangle$

(2) Find the equations for the line tangent to the curve $\vec{r}(t) = \left\langle \cos(2\pi t),\, t^3, \sqrt{t^2+3} \right\rangle$ at the point (1,-1,2) (9 points)

For a line, need a point an direction vector.

Point (1,-1,2)

direction vector is $\vec{r}\,'$ at that point. The point (1,-1,2) occurs for $t = {}^-1$ so $\vec{v} = \vec{r}\,'(1) = \langle 0, 3, {}^-\tfrac{1}{2} \rangle$

$$\boxed{\begin{cases} x = 1 \\ y = -1 + 3t \\ z = 2 - \frac{1}{2}t \end{cases}}$$

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(3) Find an equation of the plane containing the point (3,7,-9) and orthogonal to the line

$$L \begin{cases} x = 4t + 1 \\ y = 3 - t \\ z = 8 + 3t \end{cases}$$

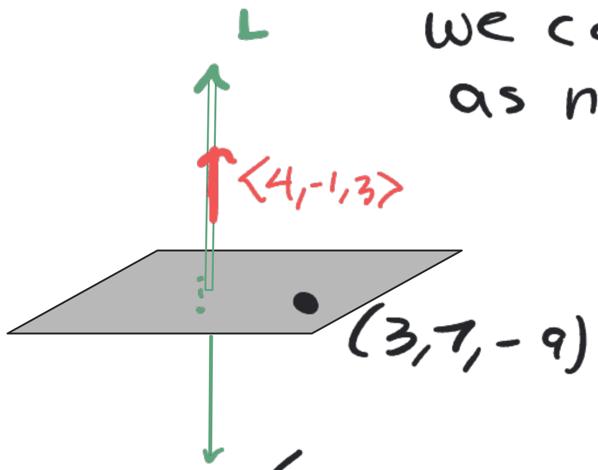
(9 points)

Plane: need point = 3,7,-9

normal $\vec{n} = \langle 4, -1, 3 \rangle$

Note: Sometimes a rough sketch (not to scale on axes, just a random point in a random plane with an orthogonal line) helps

Since line is orthogonal to plane we can use it's direction vector as normal for plane



$4(x-3) - (y-7) + 3(z+9) = 0$

$$\boxed{4x - y + 3z = -22}$$

explain or show sketch

$a \cdot b = 6 + 5 - 35 = -24$ $\qquad \|\vec{a}\| = \sqrt{30}$ $\quad \|\vec{b}\| = \sqrt{83}$

(4) Given the vectors $\mathbf{a} = \langle 2,1,-5 \rangle$ and $\mathbf{b} = \langle 3,5,7 \rangle$, find the following:

$\vec{b} = \langle 3,5,7 \rangle$

(4 points each)

a) **a** X **b** **(no partial credit here – easy to check)**

check $32(2) - 29(1) + 7(-5) = 0$ ✓

$32(3) - 29(5) + 7(7) = 0$ ✓

$\langle 32, -29, 7 \rangle$

b) the angle between **a** and **b**

$\theta = \cos^{-1} \dfrac{-24}{\sqrt{30}\sqrt{83}}$

$= \cos^{-1} \dfrac{-24}{\sqrt{2490}} \approx$

give exact answers

$\cos^{-1}\left(\dfrac{-24}{\sqrt{30}\sqrt{83}}\right)$

c) proj $_{\mathbf{b}}\mathbf{a}$

$\dfrac{\vec{a} \cdot \vec{b}}{\vec{b} \cdot \vec{b}} \vec{b} = \dfrac{-24}{83} \vec{b}$

$\left\langle \dfrac{-72}{83}, \dfrac{-120}{83}, \dfrac{-168}{83} \right\rangle$

d) a unit vector in the direction of **b**

$\vec{u} = \dfrac{1}{\|\vec{b}\|} \vec{b} = \dfrac{1}{\sqrt{83}} \vec{b}$

$\left\langle \dfrac{3}{\sqrt{83}}, \dfrac{5}{\sqrt{83}}, \dfrac{7}{\sqrt{83}} \right\rangle$

e) a value for k such that < k, 3,-6 > is orthogonal **b**

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dot product must be zero

$3k + 15 - 42 = 0$

$3k = 27$

$k = 9$

9

(5) Find equations for the line of intersection for the planes $x+2y+3z=1$ and $x-y+z=1$ (10 points)

Algebraically or Geometrically with Vector

$$\begin{cases} x+2y+3z=1 \\ x-y+z=1 \end{cases}$$

$\vec{n}_1 = \langle 1,2,3\rangle$

$\vec{n}_2 = \langle 1,-1,1\rangle$

$$\begin{cases} x+2y=1-3z \\ x-y=1-z \end{cases}$$

$\vec{v} = \vec{n}_1 \times \vec{n}_2 = \langle 5,2,-3\rangle$

point: $(1,0,0)$

(find any point on both planes)

Elim x: $3y=-2z \Rightarrow y=\frac{-2}{3}z$

find x $x=y+1-z$

$=-\frac{2}{3}z+1-z$

$=1-\frac{5}{3}z$

$$\boxed{\begin{cases} x=1-\frac{5}{3}t \\ y=-\frac{2}{3}t \\ z=t \end{cases}}$$

$$\boxed{\begin{aligned} x&=1+5t \\ y&=2t \\ z&=-3t \end{aligned}}$$

(6) Find the distance between the parallel planes $\overset{\text{plane 1}}{2x-y+3z=5}$ and $\overset{\text{plane 2}}{-6x+3y-9z-5=0}$ (10 points

Find any point on plane 1: $(1,0,1)$

Then used distance formula

$$d=\frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$$

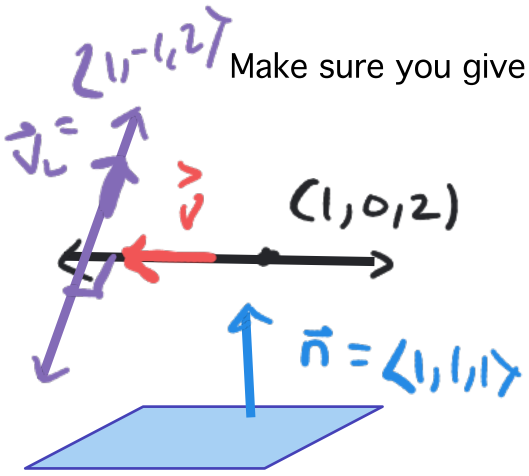
$$=\frac{|-6(1)+3(0)-9(1)-5|}{\sqrt{36+9+81}}=\frac{20}{\sqrt{126}}=$$

$$=\boxed{\frac{20}{3\sqrt{14}}}$$

(7) Find parametric equations for the line through the point (0,1,2) that is parallel to the plane

$x+y+z=2$ and perpendicular to the line , $L\begin{cases} x=t+1 \\ y=1-t \\ z=2t \end{cases}$

Make sure you give a clear explanation (10 points)



Need direction vector, $\vec{v}$ for desired line.

- Since desired line is parallel to plane, $\vec{v}$ is orthogonal to the normal to the plane $\vec{n}=\langle 1,1,1\rangle$
- Since desired line perpendicular to line L, $\vec{v}$ is orthogonal to $\vec{v}_2=\langle 1,-1,2\rangle$ from L

$$\vec{v}=\vec{n}\times\vec{v}_2=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & -1 & 2 \end{vmatrix}=\langle 3,-1,-2\rangle$$

$$\boxed{\begin{aligned} x &= 3t \\ y &= 1-t \\ z &= 2-2t \end{aligned}}$$

(8) Determine whether the line through (-3,1,0) and (-1,5,6) intersects the plane. $2x + y - z = -2$, and if so, where? (10 points)

P Q

Line $\overleftrightarrow{PQ}$

direction vector $\vec{v} = \overrightarrow{PQ} = \langle 2,4,6 \rangle$

Line $\begin{cases} x = -3 + 2t \\ y = 1 + 4t \\ z = 0 + 6t \end{cases}$

Intersect plane?

$2(-3+2t) + (1+4t) - (6t) = -2$

$-6 + 4t + 1 + 4t - 6t = -2$

$2t = 3$

$t = 3/2$ ⇒ on intersection exists

Point of intersection

$L = \begin{cases} x = -3 + 2(3/2) = 0 \\ y = 1 + 4(3/2) = 7 \\ z = 6(3/2) = 9 \end{cases}$

$\boxed{(0,7,9)}$

(9) Match the vector function (a-d) with the graph below (I-VI), The scale is purposely left off the graphs so do not assume scale. Two graphs have no match, you do not need to do anything with them. . Axes are standard orientation with x axis red, y axis green and z axis blue (8 points)

a) $\vec{r}(t)=\langle t^2, t\cos(t), t\sin(t)\rangle$ VI

b) $\vec{r}(t)=\langle t^2, t^2, t^4\rangle$ V

c) $\vec{r}(t)=\langle \sin(t), \cos(t), t\rangle$ IV

d) $\vec{r}(t)=\langle t, 2t, \sin(t)\rangle$ III

c) Helix $x=\sin t$
$y=\cos t$

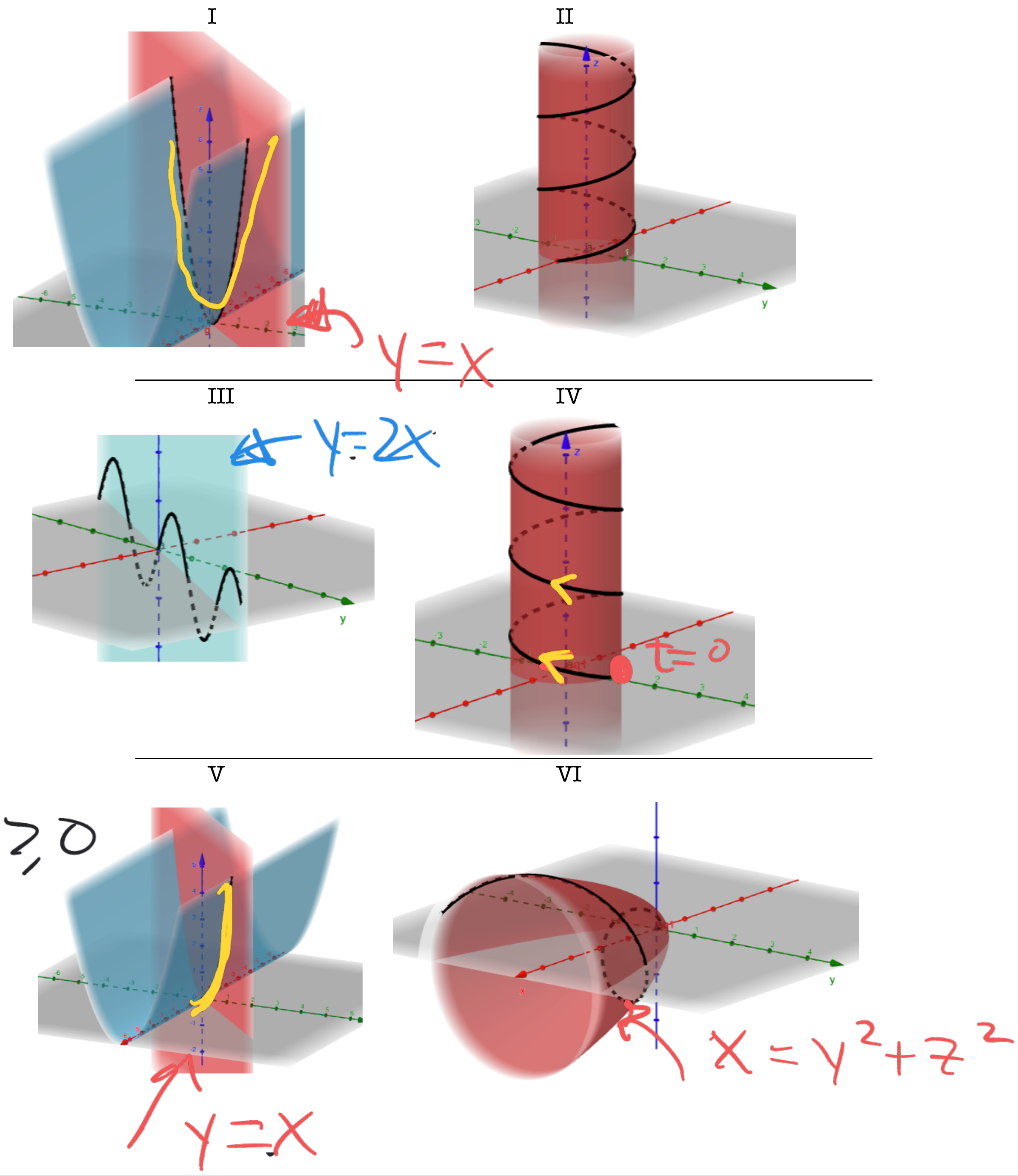
Direction

$t=0$

Clockwise from above

d) $x=t$
$y=2t \Rightarrow y=2x$

$z=\sin t$ osculates



x, y ≥ 0

a) Notice $x=t^2 \Rightarrow x \geq 0$. That rules out some choices. Then $y^2+z^2=t^2\cos^2 t+t^2\sin^2 t=t^2(\cos^2 t+\sin^2 t)$
so $y^2+z^2=t^2=x$ This is a paraboloid opening about x-axis $\boxed{VI}$

b) $x=y \Rightarrow$ on plane $x=y$ so either I or V
(Also $z=x^2$ so on that parabolic cylinder)
But since $x=y=t^2$, x,y must both be $\geq 0 \Rightarrow$ $\boxed{V}$